Marketing Mix Modelling and Return on Investment

P.M Cain

Taken from:

Integrated Brand Marketing and Measuring Returns

Edited by
Professor Philip J. Kitchen

A. Introduction

The marketing mix model is a widely-used tool to evaluate Return on Investment (ROI) and inform optimal allocation of the marketing budget. Economics and econometrics lie at the heart of the process. In the first place, the model structure is derived from microeconomic theories of consumer demand ranging from single equations of product sales to full interactive systems of brand choice. Secondly, econometric techniques are used to estimate demand response to marketing investments, separating product sales into base and incremental volume. Base sales represent the long-run or trend component of the product time series, driven by factors ranging from regular shelf price and selling distribution to underlying consumer brand preferences. Incremental volume, on the other hand, is essentially short-run in nature, capturing the week-to-week sales variation driven by temporary selling price, multi-buy promotions and above the line media activity. These are converted into incremental revenues or profits and benchmarked against costs to calculate ROI to each element of the marketing mix.

Focusing solely on incremental volumes in this way implies that conventional marketing mix models provide insight into short-term ROI only. As such, they often lead to marketing budget allocations biased towards promotional activity: short-run sales respond well to promotions, yet are less responsive to media activity – particularly for established brands. This, however, ignores the longer-term perspective: that is, the potential brand-building effects of successful media campaigns on the one hand and the brand-eroding impacts of heavy price discounts on the other. Acknowledging and quantifying these features is crucial to a complete ROI evaluation and a more strategic budget allocation.

Measuring the long-run impact of marketing investments requires a focus on the base sales component of the marketing mix model. This is simply because any long-term brand-building effects reside in the level or trend component of the sales series and impact the evolution in base sales over time. The ability to uncover these effects depends crucially on the data and choice of econometric methodology used. The conventional approach uses static Ordinary Least Squares (OLS) techniques which impose a fixed or deterministic baseline. Not only can this give an artificial split into

base and incremental volumes, it precludes any analysis of the long-run impact of marketing activity by construction. One solution is to apply the dynamic cointegrating Vector Autoregression (VAR) model, an estimation technique commonly used in the econometrics literature for evaluating the long-term effects of economic indicators. Examples in the marketing literature can be found in *inter alia* Dekimpe *et al* (1999). In practice, however, this technique is often impractical in the context of fully specified mix models. A preferable approach is to use a methodology that can directly separate both the short and long-run features of the data − allowing a complete analysis of both in distinct stages. Time series regression analysis is a logical choice for two reasons. Firstly, all marketing mix models involve time-ordered data and are essentially time series equations with additional marketing mix components. Secondly, the technique provides a direct decomposition of any time-ordered data series into a trend, seasonal and random error component. It is then a natural step to decomposition of product sales into short-term marketing factors (incremental) and long-term base (trend). This generates an evolving baseline, which can then be meaningfully analysed to quantify long-run ROI.

In this chapter, we develop these issues in detail. Section B outlines the foundations of the marketing mix model. Against the background of the conventional approach, we put forward alternative theoretical and econometric frameworks for improved short-term ROI evaluation. The section is then completed with a technique for evaluating the long-term effects of marketing investments and how these may be combined with short-term results to provide total ROI. Section C discusses the managerial benefits of the mix model and the total returns on marketing. Section D concludes.

B. The modelling process

Evaluating total marketing ROI proceeds in five key stages illustrated in Figure 1, ranging from the underlying economic model and level of analysis through to quantification of the long-term impact of marketing investments on sales and profits.

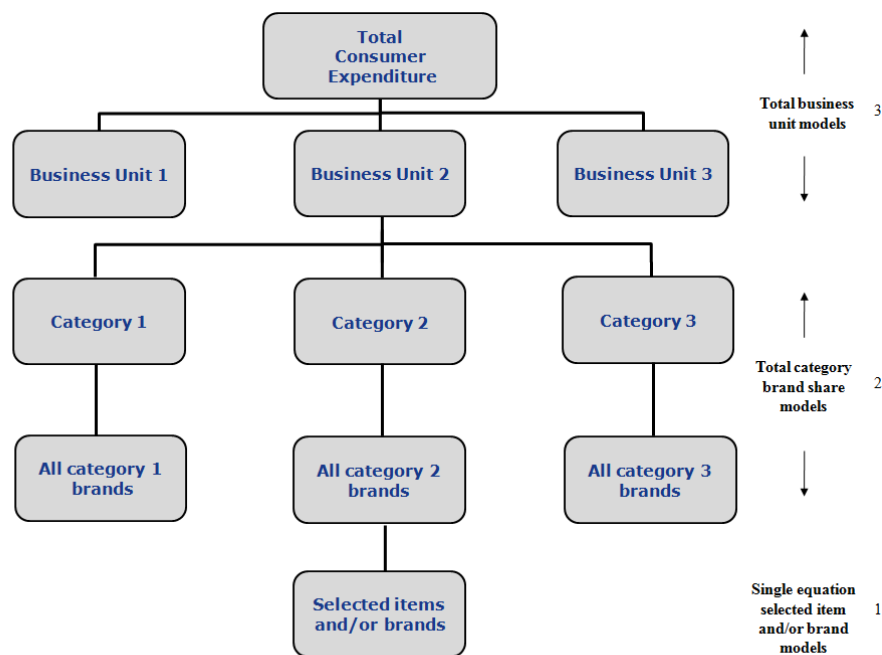**Figure 1: The marketing ROI modelling process**
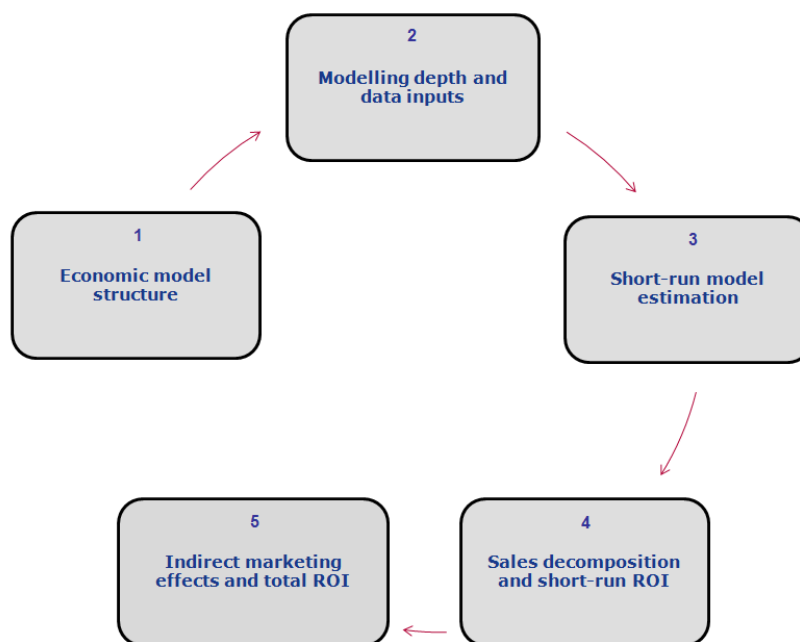
Steps 1-4 represent the key ingredients of short-term marketing ROI evaluation. Together, they comprise the basic framework of the standard marketing mix model. Two points are worthy of note here. Firstly, conventional approaches tend to overlook the microeconomic consumer demand structures underlying the model form. However, it is important to be aware of these so the best model(s) can be chosen for any particular situation. Secondly, econometric estimation of the model parameters tends to follow a standard OLS regression route – with little attention paid to the fundamental time series nature of the data involved. This is unfortunate, as considerable information can be lost leading to inaccurate short-term ROI measurement. Time series estimation techniques that accurately model both the short and long-run components of the data are preferable. Step 5 completes the process, outlining an approach to long-run ROI measurement where consumer tracking research is merged with outputs from the short-term marketing mix model.

1) Economic model structure

All marketing mix models are based on microeconomic models of product demand with a view to inferring consumer response to each element of the marketing mix. Consequently, an important starting point is the type of consumer demand model used. Figure 2 presents a stylised structure of a product hierarchy designed to illustrate the various options available.

Figure 2: A modelling hierarchy of products, brands and business units

Total
Consumer
Expenditure

Total business
unit models   3

Business Unit 1   Business Unit 2   Business Unit 3

Category 1   Category 2   Category 3

Total category
brand share   2
models

All category 1
brands   All category 2
brands   All category 3
brands

Selected items
and/or brands

Single equation
selected item   1
and/or brand
models

Each level in Figure 2 corresponds to a specific model of consumer demand at different degrees of product aggregation. Level 1 comprises single equation models run at individual product level, or aggregated into similarly priced and promoted groups or 'items'. If the aim is to gain a picture of total brand performance, sets of single equations at product or item level can be estimated. Alternatively, the data can be aggregated into variants or a single brand variable. To gain a coherent picture of total category demand, on the other hand, we proceed to Level 2 models. Similarly, these too may comprise sets of product level equations. However, the number of required equations is often prohibitive. Consequently, products are generally aggregated to items, variants or total brand level. As we move to Level 3, models of total business units usually involve highly aggregated sales, marketing and macroeconomic data.

1.1) Single equation models

The conventional approach to modelling the marketing mix focuses on selected items and/or brands in the manufacturer's portfolio of products. This 'single-equation' approach generally uses the following type of demand model:

$$S_{it} = \exp(\alpha_i + T_i + \delta_i + \varepsilon_{it}) \prod_{j=1}^{n} \prod_{k=1}^{M} f_m(X_{kit})^{\beta_{ijk}} \qquad (1)$$

Which stipulates that sales of product i (S_i) over time t are a multiplicative function (f_m) of a set of marketing and economic driver variables X_{kit}[1]. The demand equation is completed with an intercept α_i, trend (T_i), seasonal index (δ_i) and an error term ε_i. The intercept is equal to the mean of the sales data - net of the parameter weighted means of the explanatory variables - and equivalent to the expected level of non-marketed product sales. This is often referred to as *base* sales. The trend term caters for any observable 'drift' present in the base over time and the seasonal index caters for regular 'time-of-year' factors and period-specific holiday effects. The error term represents all unexplained factors influencing demand and must satisfy the usual properties of classical regression in order for us to interpret the demand parameters with confidence and precision. As it stands, equation (1) is non-linear. For the purposes of estimation, the model is converted into an additive form by taking natural logarithms thus:

$$LnS_{it} = \alpha_i + T_i + \delta_i + \sum_{j=1}^{n} \sum_{k=1}^{M} \beta_{ijk} \ln X_{kit} + \varepsilon_{it} \qquad (2)$$

The log-linear single-equation level 1 approach is adequate if we wish to focus on single products at a time and is a popular choice due to the fact that estimated parameters β_{ijk} are immediately interpretable as demand elasticities. However, it does suffer from two key drawbacks.

---

[1] Standard (additive) linear forms are also used. However, the multiplicative model is often chosen due to the implied relationships between demand and the chosen explanatory variables. For example, demand drops exponentially to zero as price approaches infinity and advertising exhibits diminishing returns as weight increases. These, together with the implied synergies between the variables, are usually deemed desirable properties.

Firstly, it is well known that the double logarithmic functional form is inconsistent with the adding up constraint of conventional microeconomic demand theory (Deaton and Muellbauer, 1980). Equation (2) is often applied in practice to several competing products in one group or 'category'. Each equation for product i is specified as a function of $k = 1...M$ of its own marketing mechanics together with $j = 1...n$ of the (competitive) marketing drivers of the other items in the group. Problems arise when the estimated volume steal from product i due to the marketing activity X_j of competitor products does not match total competitor volume gains. That is, it is perfectly possible that volume steal is either less than or greater than volume gains. Whereas this is usually interpreted as category growth or shrinkage respectively, it is simply a consequence of the fact that sets of single equations are unrelated to each other and do not 'add-up', telling us nothing about genuine category effects of product marketing.[2]

Secondly, marketing incremental is a gross figure: that is, each model delivers a total amount of incremental volume to each element of the marketing mix. However, we cannot accurately define the source of this incremental: specifically, how much is due to substitution from other brands and how much is due to category expansion effects?[3]

---

[2] Violation of the adding up property refers to the fact that single-equation functional forms such as (2) give distorted expenditure patterns across the modelled products that are greater or less than total expenditure across the group.

[3] This problem can be partly alleviated by specifying marketing effects in relative terms. For example, price in each demand equation can enter in absolute and relative terms providing an elasticity decomposition into two components: a relative demand response when price of good i changes relative to all other goods in the group and a matched demand response when prices of all products in the group move together. Although analogous to a separation of substitution and category level effects, this approach is still inconsistent with the adding up constraint.

1.2) Total category models

To overcome these problems, simultaneous equation demand system approaches are required. There are several theoretical structures that can be used. On the one hand, we have continuous choice models such as Stone's Linear Expenditure system (Stone, 1954), the Rotterdam model (Theil, 1965, Barten, 1966) and the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980).[4] On the other hand, there are discrete choice approaches such as the attraction models illustrated in Nakanishi and Cooper (1974). Here we focus on the attraction model due to its more widespread use in the marketing literature. The functional form of the model is based on the notion that attractiveness of product i in a chosen category is a function of k marketing efforts X_k. This gives:

$$A_i = \exp(\alpha_i + T_i + \delta_i + \varepsilon_i) \prod_{j=1}^{n} \prod_{k=1}^{M} f_m(X_{kit})^{\beta_{ijk}} \tag{3}$$

The volume share of product i as a proportion of total category volume sales is then defined as equal to its share of attractiveness out of total attraction of the category. Thus we have:

$$s_i = \frac{A_i}{\sum_{j=1}^{n} A_j} \tag{4}$$

where A_i represents the attractiveness of brand i and $\sum_{j=1}^{n} A_j$ represents the total attractiveness of the category summed over all n brands. Substituting equation (3) into equation (4), gives us the general form of the sum-constrained market share model where all product shares sum to unity. As for the single equation approach, the resultant model form is non-linear and must be transformed in order to provide an estimable functional form. Taking logarithms of both sides of (4) and subtracting the pth product share in the system we have:

$$\ln(s_i) - \ln(s_p) = \ln(A_i) - \ln(A_p) \tag{5}$$

---

[4] See Cain (2005) for an application of the dynamic AIDS model.

Substituting (3) into (5) gives:

$$\ln\left[\frac{s_{it}}{s_{pt}}\right] = \left[\alpha_i - \alpha_p\right] + \left[T_i - T_p\right] + \left[\delta_i - \delta_p\right] + \sum_{k=1}^{M}\sum_{j=1}^{n}(\beta_{kij} - \beta_{kpj})\ln X_{kjt} + \left[\varepsilon_i - \varepsilon_p\right] \quad (6)$$

Equation (6) gives the general form of the sum-constrained log-ratio demand system, which predicts the (aggregated) probability of product choice from a consumer consideration set in terms of given marketing driver variables.[5] The model can be written as a set of *n-1* reduced form log-ratio share equations, each as a function of product specific marketing effects β_{ki} for each marketing mix variable and a full set of direct competitor cross effects β_{kj}.[6] The pth numeraire share equation is defined by the model adding up constraint − which is used to derive the underlying (structural) parameters of all *n* shares.

Data limitations and collinearity issues usually preclude estimation of model (6) as it stands. It is more usual to consider restricted versions of the model nested within this general extended form. Firstly, the *differential* effects model constrains all direct cross effects β_{kj} to zero − but allows product specific marketing effects β_{ki}. Secondly, the *constant effects* model goes further and imposes equal response effects β_k for all competing products. Both models can be seen as sets of cross equation parameter restrictions on the general model. Any combination of data admissible restrictions is feasible − leading to combinations of constant and differential effects model.[7]

Finally, since the specification of the attraction model is in log-ratio share form, we need to ensure that the marketing response estimates are invariant to the choice of pth numeraire share. Furthermore, the system requires a model for total category volume

---

[5] The log-centered form of the attraction model is fully discussed in Nakanishi and Cooper (1974). The log-ratio approach used in the text provides equivalent parameter estimates and is easier to work with (see Houston *et al.* 1992).

[6] Note that the parameter estimates of (9) are reduced form in that they are a composite of structural and residually defined parameter estimates of the pth product.

[7] The Multiplicative Competitive Interaction (MCI) and Multi-Nomial-Logit (MNL) forms of the attraction model are based on a logit choice structure and derived from the consumer's utility function − where the relative odds of choosing product *i* over *j* is the same, no matter what other alternatives are available. This embodies the Independence of Irrelevant Alternatives (IIA) property which implies that substitution between products in response to marketing mix changes is in proportion to market share. This property is only true for the constant effects and differential model forms. Incorporating combinations of direct cross effects allows us to test deviations away from this basic competitive structure, where substitution may also be related to similarity of product characteristics.

such that we can derive equations for product volume. The former is ensured by maximum likelihood estimation of the system.[8] The latter is dealt with by a category volume equation such as:

$$\ln CV_t = \alpha + T_i + \delta_t + \sum_{j=1}^{n}\sum_{k=1}^{K} \rho_{ij} \ln X_{kj} + \sum_{L=1}^{l} \phi_i Z_t + u_t \tag{7}$$

Equation (7) is estimated in terms of relevant brand marketing variables X_{jt}, a range of macroeconomic drivers Z_t, trend and seasonal components. Together system (6) and equation (7) give a set of product volume equations which deal with both of the problems raised in the single-equation approach. Substitution effects and share elasticities are derived through the share model and the impact of product specific marketing effects on the category is estimated through the category model.[9]

---

[8] Since each equation in system (6) contains the same set of explanatory variables, OLS regression estimation automatically satisfies adding up – and parameter estimates are invariant to choice of *numeraire*. As soon as we move to the constant or differential effects form, or combinations thereof, we are imposing cross-equation zero restrictions on the model structure leading to a non-diagonal error covariance matrix. Maximum likelihood estimation is then required to ensure invariance (Barten, 1969).

[9] More flexible category expansion effects can be derived via inclusion of an outside good directly into the share system – with no need for a total category volume model. However, specification of the outside good is contingent on estimated potential market size and can be problematic.

1.3) Business unit models

It is rare for manufacturers to operate in just one category. The manufacturing portfolio generally comprises broad ranges of products across different categories known as business units. For example, categories such as detergents, soaps, deodorants, oral and skin care may be grouped under a wider business unit label such as Health and Personal Care. A 'bottom up' picture of total business unit marketing effectiveness may be obtained by aggregating results for all relevant manufacturer brands across a range of modelled categories. However, this generally requires a considerable number of models. An alternative 'top down' approach is to run models directly at a business unit level, with data aggregated across each manufacturer's portfolio of categories. A useful economic model at this highly aggregated level is the AIDS model of Deaton and Muellbauer (1980) illustrated in equations (8) and (9).

$$S_{BUit} = \alpha_i + T_i + \sigma_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \sum_{m=1}^{M} \sum_{j=1}^{n} \phi_{mij} X_{mt} + \beta_i \ln\left[\frac{TCE_t}{\tilde{p}_t}\right] + \varepsilon_{it} \qquad (8)$$

$$LnTCE_t = \alpha + T + \delta_i + \sum_{l=1}^{L} \phi_i Z_{lt} + u_t \qquad (9)$$

Equation (8) indicates that the revenue share of total business unit sales for manufacturer *i* out of total consumers' expenditure (*TCE*) on categories across all manufacturers playing in the same business unit is a function of an index of aggregate prices and marketing variables (X_m), together with trend, seasonal index and deflated total consumers' expenditure.[10] Equation (9) specifies that *TCE* itself is a function of a range of macroeconomic variables Z_t. Together, equations (8) and (9) make up a joint model of aggregate consumer demand for each manufacturer's business unit, where shifts in overall macroeconomic activity, and their impact on business unit performance, feed naturally from (9) to (8).

---

[10]Note that the dependent variable in this model is value share – as opposed to volume share in the category model structure of Section 1.2. This is a consequence of the alternative microeconomic foundations underlying the model. The functional form of the AIDS model is derived from the classic consumer utility maximisation problem: namely, the choice of optimal quantities of goods demanded subject to a fixed money income constraint. As such it is a continuous choice model since consumers demand optimal quantities of *all* goods in the system. This can be applied at many levels of aggregation but is best suited to highly aggregated data sets where it is natural to invoke the concept of the aggregate consumer.

The three levels of market mix modelling illustrated in this section highlight two key issues: namely, the microeconomic origins of alternative types of model on the one hand and how ROI analysis can be pitched at varying degrees of product aggregation on the other. It is important to recognize, however, that each model structure is not limited to each level of aggregation. For example, single equation volume models can be applied at any level of the hierarchy – they just lack the microeconomic consistency of the demand system structures. Demand system approaches may also be used for competing groups of individual product lines, but require many component equations to arrive at a full category scope. Combinations of both are also possible, providing an integrated model of the manufacturer portfolio. For example, demand system models can be used to gain a complete picture of the total category at brand level. Single equation approaches can then be used to drill down into the product detail contained in each brand aggregate. In sum, a wide variety of options are possible, with the types and combinations of models used tailored to relevant business issues.

2) Modelling depth and data inputs

The economic models of Section 1 can be applied, in principle, across a range of industries, as diverse as Fast Moving Consumer Good (FMCG) to financial and automotive. In practice, however, model choice is heavily dependent on available data and relevant business issues. For example, full category data are often unavailable to individual players in sectors such as financial services, thus leaving conventional single equation modelling as the only option. On the other hand, such data are generally available in the Fast Moving Consumer Good (FMCG) or automotive sectors, thus allowing applications of simultaneous equation category modelling. Once the scope of available data has been determined and the relevant economic models identified, the next step is to specify the modelling 'depth' and driver variables involved.

2.1) Modelling depth

Product sales at each level of the hierarchy in Figure 2 constitute 'revealed' consumer demand for the product aggregate in question. Models at each level may also be run at differing levels of aggregation across the consumer as illustrated in Figure 3.

Figure 3: Levels of consumer aggregation
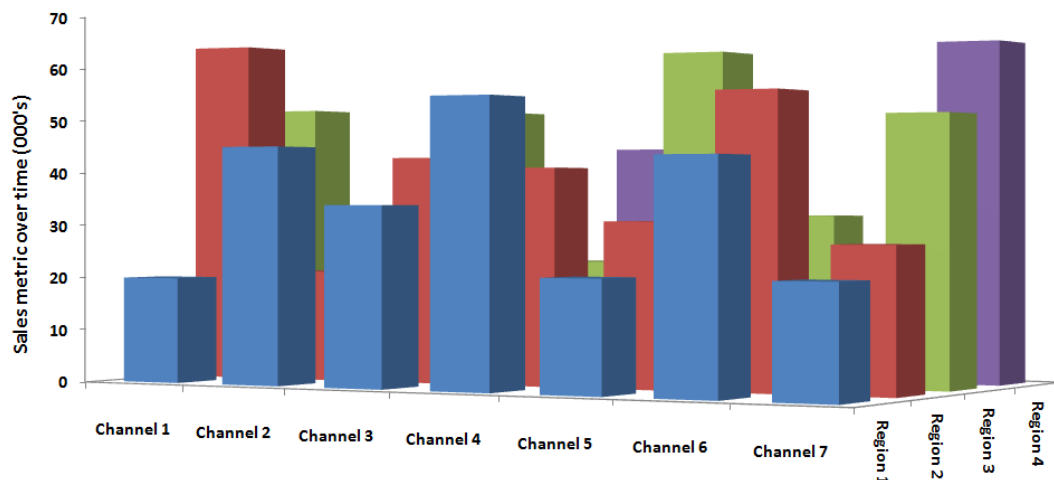
The left hand side axis represents the sales metric over time at the chosen level of the modelling hierarchy outlined in Section 1. The bottom axis represents the sales channel or level of consumer aggregation: households, stores, groups of stores (chains or key accounts) up to total market level. The right hand side axis indicates how these groups may also be split by geographical region. Each block thus depicts a time series for each regional-channel combination, designed to demonstrate how the models of section 1 can be estimated across different cross-sectional units – where increased parameter precision can be gained from pooling time-based observations across different dimensions. Such models use longitudinal panel data sets and are known as Time Series Cross Section (TSCS) models. Household level panel models represent the lowest level of consumer aggregation and tend to reside solely in the academic domain, with many examples in the literature (*inter alia* Jedidi *et al* 1999). Moving up the aggregation tree takes us first to store and then to key account panel models across regions and/or market level. Such models are readily offered on a commercial basis. Pure time series based models, involving single time series at market level, represent the highest level of consumer aggregation and often offered where limited cross-sectional data are available.

2.2) Demand drivers

Demand driver variables are chosen to represent the full marketing mix, ranging from selling distribution, price promotion, multibuy and display activity through to TV, press, magazine, radio and internet investments. Media data are also often split up into separate campaigns to isolate differential effectiveness by message. Diminishing returns for increasing media weight are implicitly incorporated into the multiplicative form of equation (1). However, this is often augmented to incorporate non-constant elasticities to test for additional saturation effects. External drivers such as macroeconomic data on consumers' income, GDP and interest rates can also be used and generally sourced from government statistical departments. However, their use will depend on the level of product aggregation used. The minimum level at which such data will appear is in Level 1 models using highly aggregated sales metrics, Level 2 category models such as equation (7) or Level 3 models such as equation (9). It is highly unlikely that such data will feature as significant drivers in models for individual products at Level 1.

14

3) Short-run model estimation

The next step in the process is quantification of the sales response to variation in each of the marketing mix investments. This is where econometrics enters the picture: a statistical regression based procedure to estimate the parameters of the theoretical demand functions outlined in Section 1, at the appropriate depth outlined in Section 2. It is important to recognize, however, that such demand functions, as they stand, are inadequate for estimation purposes. Firstly, they depict contemporaneous relationships between sales and marketing variables: this is as far as economic theory will take us in deriving an estimating form for the marketing mix model, but tells us nothing about the dynamics of sales adjustment to changes in the marketing mix. Consequently, it is implicitly assumed that consumers adjust immediately to changes in the driver variable(s). Secondly, underlying brand tastes are not modelled in any way. A time trend is often added to the intercept of the static functional forms of Section 1 to allow for a general drift in tastes over time, but this is purely deterministic and user-imposed. It tells us nothing about genuine long-term evolution in the sales data.

Both of these issues indicate that more flexible dynamic forms of the mix model are required for estimation purposes. This section focuses on two complementary specifications: autoregressive distributed lag (ADL) models and time varying parameter (TVP) models. ADL models improve the underlying *behavioural* relationship between sales and marketing by explicitly recognising that consumers take time to fully adjust to changes in marketing investments - due to factors such as brand loyalty, habit formation and short to medium-term repeat purchase. TVP models, on the other hand, address the underlying *time series* properties of the sales series, recognising that sales also adjust to longer-run factors such as changes in brand tastes, potentially driven by a host of factors in which marketing may or may not play a role.

3.1) ADL models

A general dynamic functional form rewrites the static model as an Autoregressive Distributed Lag (ADL) function in terms of lagged sales together with current and past driver values. In the case of the single equation model (2), we have:

$$LnS_{it} = \alpha_i + T_i + \delta_i + \sum_{j=1}^{n}\sum_{k=1}^{M}\sum_{l=1}^{T} \beta_{ijk} \ln X_{kit-l} + \sum_{l=1}^{T} \lambda \ln S_{it-l} + \varepsilon_{it} \qquad (10)$$

This is known as a dynamic relationship and is intended to capture the short to medium term effects of marketing investments as consumers adjust to new levels of the relevant driver(s). Analogous specifications can also be applied to demand system approaches. However, the presence and final lag length (l) of the X_k and sales variables is purely data driven: *a priori* economic theory does not tell us precisely how consumers adjust. Nevertheless, many hypothesised forms of adjustment are nested within this general functional form, which dictate the ultimate shape of the adjustment path. This section considers three such models.

a) Adstock

The Adstock concept was introduced into the marketing literature by Broadbent (1979) and represents the conventional approach to incorporating dynamics into the marketing mix model – applied solely to TV advertising. The idea is intended to capture the *direct* current and future effects of advertising, where a portion of the full effect is felt beyond the period of execution due to the interplay between media retention and the product purchase cycle. This is equivalent to a restricted form of the distributed lag model – nested in equation (10) – with lags (l) applied solely to the advertising (TVR) variable:

$$LnS_{it} = \alpha_i + T_i + \delta_i + \sum_{l=0}^{T} \beta_{l+1} \ln(TVR_{t-l}) + \sum_{j=1}^{n}\sum_{k=2}^{M} \beta_{ijk} \ln X_{kit} + \varepsilon_{it} \qquad (11)$$

Assuming that the distributed lag coefficients decline geometrically, we may write:

$$LnS_{it} = \alpha_i + T_i + \delta_i + \beta(1-\gamma)\sum_{l=0}^{T} \gamma^l \ln(TVR_{t-l}) + \sum_{j=1}^{n}\sum_{k=2}^{M} \beta_{ijk} \ln X_{kit} + \varepsilon_{it} \qquad (12)$$

The parameter γ represents the appropriate retention rate, the value of which is bounded between 0 and 1, usually chosen via a search procedure to maximize the in-sample fit of the model: longer retention rates are indicative of a higher quality of advertising and/or a shorter consumer purchase cycle. This approach represents a concise way of encapsulating a distributed lag effect in one simple variable. Equation (12) represents the basic functional form of the bulk of marketing mix analyses, with demand parameters estimated using OLS regression techniques on panel or market level time series data.[11]

b) Partial adjustment

An alternative dynamic specification assumes that consumers partially adjust towards a desired or equilibrium sales level following a change in the marketing variables. This theory gives the following functional form:

$$LnS_{it} = \alpha_i^* + T_i^* + \delta_i^* + \sum_{j=1}^{n} \sum_{k=1}^{M} \beta_{ijk}^* \ln X_{kit} + \lambda^* \ln S_{t-1} + \varepsilon_{it} \qquad (13)$$

where the parameter λ^* measures the rate of adjustment towards equilibrium.[12] As for the Adstock model, this is simply a restricted form of the full ADL specification. However, an implicit lag structure now appears via the lagged sales term. Consequently, the dynamic interpretation is different. Forms like (13) are often used to model direct current period marketing effects, where dynamic 'carryover' effects work *indirectly* through repeat purchase based on product performance.[13]

---

[11] The declining geometric lag restriction is just one possibility. Alternative (finite) lag structures may be imposed such as Polynomial Distributed lags (Almon, 1965), which allow tests of advertising 'wear-in', where maximum response to advertising can occur after the period of deployment.

[12] Estimated parameter λ^* equals (1-ϕ), where the adjustment rate ϕ plays the same role as the retention rate γ in equation (12). The closer ϕ is to one, the more adjustment is immediate. As ϕ approaches zero, so consumers take longer to adjust and marketing impacts are 'stretched' over a longer time horizon.

[13] An alternative approach is to apply an infinite distributed lag structure to the explanatory variable(s) in equation (2) and apply a Koyck transform (Koyck, 1954). This gives an observationally equivalent form to (13) with a moving average error term. Both the partial adjustment and Koyck forms have been extensively examined in the marketing literature (*inter alia* Clarke 1976, Mela *et al* 1997). Note, however, that the lagged sales term captures a general persistence applying common dynamics to all variables in the model. Consequently, it is not possible to isolate the differential dynamic effects due to each marketing mechanic.

c) Error correction

An alternative re-writing of the ADL model (10) is in error correction form. For example a one-period lag with one explanatory variable X_k may be written as:

$$\Delta S_{it} = \alpha_i + T_i + \delta_i + \beta_1 \Delta X_{kt} - (1-\lambda)\left[S_{it-1} - \tilde{\beta}X_{kt-1}\right] + \varepsilon_{it} \qquad (14)$$

Which expresses the change in sales in terms of the change in X_k and the lagged levels of sales and X_k. Forms like (14) go one step further than (13). Rather than simply positing the existence of a desired level of product demand, the model hypothesises that an equilibrium relationship exists between the levels of X_k and sales. For example, suppose that the manufacturer strategy is to maintain a certain ratio between sales and TV advertising expenditure – a ratio determined by the parameter $\tilde{\beta}$. An increase (decrease) in the level of advertising activity shifts the sales-TV relationship away from its strategic long-term ratio: that is, sales and advertising are knocked out of 'equilibrium'. Sales first increase (decrease) immediately by a factor β_1, followed by a further feedback increase (decrease) in sales to restore the underlying ratio. Analogously to equation (13), the rate of adjustment towards the equilibrium ratio is determined by the parameter *(1-λ)*. In this way equation (14) captures both the short-run per period changes in sales due to advertising together with the medium term adjustment in sales over subsequent time periods as equilibrium is restored. A good example can be found in Baghestani (1991).

The most common dynamic specification used in marketing mix applications is the Adstock form (12), which is sometimes augmented with a lagged sales variable as in equation (13) to add a purchase feedback effect.[14,15] All such forms, however, retain the fixed intercept and trend component of model (2). Product demand is thus assumed to fluctuate around a constant level and deterministic drift factor (if present) and not permitted to adjust in line with evolving product tastes over time. With no

---

[14] This is somewhat *ad hoc* as it can be argued that one partially double-counts the other: that is, the Adstock form is a truncated form of the Koyck model – which itself is observationally equivalent to the purchase feedback model. Separating carryover and purchase reinforcement effects requires an alternative theoretical specification. See Givon and Horsky (1990).

[15] The error correction model is rarely used in practice since the dynamics require relationships between continuous variables - as illustrated in Baghestani (1991). The level error correction term in (14) cannot incorporate the short to medium term dynamics of discrete variables such as advertising TVR data and temporary price cuts frequently used in marketing mix studies. Regular shelf price and selling distribution are generally the only conventional continuous regressor variables capable of forming error-correcting relationships with sales.

allowance for systematic variation in product tastes, the potential for persistent or long-term effects of marketing is, therefore, overlooked.[16]

There are two ways of addressing this. In the first place, one can simply test for an evolving taste component in the sales data using conventional unit root tests and apply non-stationary forms of the error correction model (14). The mix model would then be estimated in first differenced form using the cointegrating Vector Autoregression (VAR) approach to econometrics (Johansen 1996, Juselius 2006), potentially involving equilibrium relationships between sales and other evolving variables. However, this approach is complex and impractical in the context of fully specified models where many variables are involved. Furthermore, the autoregressive based unit root tests of Dickey and Fuller (1981) have low power against a deterministic trend alternative and, by ignoring any moving average structure in the data, display poor statistical properties (Schwert, 1987).

A preferable and more practical alternative is to allow the underlying sales level to evolve as an explicit component of the mix model. This is the approach taken by Time Varying Parameter econometrics, allowing us to extract any evolutionary taste component directly from the sales data, accurately separating short and long-run variation. The short-run component provides the basis of marketing ROI as in the standard mix model. The long-run component, on the other hand, allows us to assess the determinants of systematic long-term evolution in a second step.

---

[16] This essentially assumes that sales are stationary. Deterministic trends, such as in equation (10), impose an arbitrary view of taste evolution, where sales are essentially assumed to be trend stationary.

3.2) TVP econometrics

Marketing mix models at any level of the hierarchy in Figure 2 involve time-ordered sales observations. Consequently, they are essentially time series regressions with additional marketing driver variables – and should be estimated as such. Time series regression analysis is a statistical technique that decomposes the behaviour of any time-ordered data series into a trend, seasonal and random error component. The trend component represents evolution in the level of the sales series and is crucial to a well-specified marketing mix model. In the conventional static or dynamic approach of equations (2) and (12), this is dealt with by the regression intercept plus a linear deterministic (negative or positive) growth factor. However, trends in sales or business data rarely behave in such a deterministic fashion. Many markets, ranging from Fast Moving Consumer Good (FMCG) to durables, exhibit trends which evolve and vary over time indicative of shifts in various factors ranging from regular price and selling distribution to brand perceptions.[17] To accommodate this, the basic regression models of section 1 need to be re-cast in a more flexible time series form. Using the single equation model (2) as an example, the general form is as follows:

$$LnS_{it} = \mu_{it} + \delta_{it} + \sum_{j=1}^{n} \sum_{k=1}^{n} \beta_{ijk} \ln X_{kit} + \varepsilon_{it} \qquad \text{2(a)}$$

$$\mu_{it} = \mu_{it-1} + \lambda_{it-1} + \eta_{it} \qquad \text{2(b)}$$

$$\lambda_{it} = \lambda_{it-1} + \xi_{it} \qquad \text{2(c)}$$

$$\delta_{it} = -\sum_{j=1}^{p-1} \delta_{it-j} + \kappa_{it} \qquad \text{2(d)}$$

$$\beta_{ikt} = \beta_{ikt-1} + \upsilon_{it} \qquad \text{2(e)}$$

Equation (2a) replaces the intercept α_i in equation (2) with a time varying (stochastic) trend μ_{it}, comprising two components described by equations 2(b) and (c). This is known as the local linear trend model (Harvey, 1989). Specifically, equation 2(b) allows the underlying sales level to follow a random walk with a growth factor λ_i, analogous to the conventional trend term T_i. Equation (2c) allows λ_i itself to follow a

---

[17] See Dekimpe and Hanssens (1995) for a survey.

random walk. The variables η_{it} and ξ_{it} represent two mutually uncorrelated normally distributed white-noise error vectors with zero means and covariance matrices $\sum{}^2_\eta$ and $\sum{}^2_\xi$. Several dynamic structures are encompassed within this general specification. For example, if both covariance matrices are non-zero, the level of product demand follows a random walk with stochastic drift. If $\sum{}^2_\eta \neq 0$ and $\sum{}^2_\xi = 0$, the drift component is deterministic. If $\sum{}^2_\eta \neq 0$ and the growth factor is zero then the levels follow a random walk without drift. If both covariance matrices are zero, the data are trend stationary and the model collapses to a standard static OLS model with deterministic drift. In this way, the system can accommodate both stationary and non-stationary product demand allowing the data to decide between them. Equation 2(d) specifies seasonal effects, which are constrained to sum to zero over any one year to avoid confusion with other model components. Stochastic seasonality is allowed for using dummy variables, where p denotes the number of seasons per year, δt is the seasonal factor corresponding to time t and κ_{it} is a random error with mean 0 and covariance matrix σ^2_κ. If the latter is zero, then seasonality is deterministic. Finally, equation 2(e) allows the regression parameter for marketing variable k to evolve over the sample with a random error υ_{it} with mean 0 and covariance matrix σ^2_υ.

The dynamic time series formulation can be applied to models at all levels of the hierarchy in Figure 2 and provides a fully flexible framework for the market mix model with several key benefits. In the first place, product demand is directly decomposed into long-term and short-term components. Specifically, μ_{it} in equation 2(b) measures long-run changes in demand arising through shifts in underlying consumer tastes, leaving the β_{ijk} parameters of equation 2(a) to accurately measure short-run demand changes due to current period marketing activity. The result is a more realistic split into base and incremental volumes and more accurate short-term ROI calculation.

Secondly, the framework can accommodate the conventional behavioural dynamics outlined in Section (3.1). For example, advertising distributed lag effects can be incorporated in equation 2(a) in the form of a conventional Adstock variable as in

equation (12). Improved short and medium-term dynamic specification here provides a cleaner read on the long-term evolving component of the sales series.[18]

Thirdly, the framework naturally incorporates dynamic evolution in marketing response effects by adding additional time series equations for the response parameters β_{ij}. This is particularly important if we wish to test for shifts in marketing efficiencies over time – such as evolution in promotional and regular price elasticities for example.

Finally, having isolated the short-term impact of marketing, the extracted trend component μ_{it} allows us to build auxiliary models that focus specifically on the causes of longer-term adjustment. These can range from the impact of regular selling price, distribution and exogenous demand shocks, through to long-run effects of marketing activities. This step lies at the heart of total marketing ROI evaluation and is developed fully in Section 5 below.

---

[18] Specifying the underlying demand level to evolve as a random walk essentially caters for situations where the explanatory variables cannot fully explain the level of sales. Since the random walk form contains elements at all frequencies, it can also reflect missing short and medium term information. For example, promotional lags measuring post-promotional dips will be reflected as downward shifts in μ_i.

4) Sales decomposition and short-run ROI

4.1) Single equation approach

Estimated marketing response parameters are generally used to decompose product sales into base and incremental volume. Base sales reflect the underlying trend in the data, indicative of long-run consumer product preferences. An application of the single equation time series model 2(a)-(e), with a distributed lag advertising variable covering two national TV media campaigns, is illustrated in Figure 4 depicting the sales pattern of an FMCG face cleansing product. All short-term sales variation is clearly explained in terms of advertising, average price cuts, promotional and incremental selling distribution, features, competitive activity and seasonal demand. Base sales evolve slowly over the sample, settling from mid 2005 to mid-2006, increase until late 2006 and decline gently for the rest of the period. Total incremental volume is then used to calculate the percentage of sales volume explained by each of the sales drivers.

Figure 4: Sales decomposition with evolving baseline*
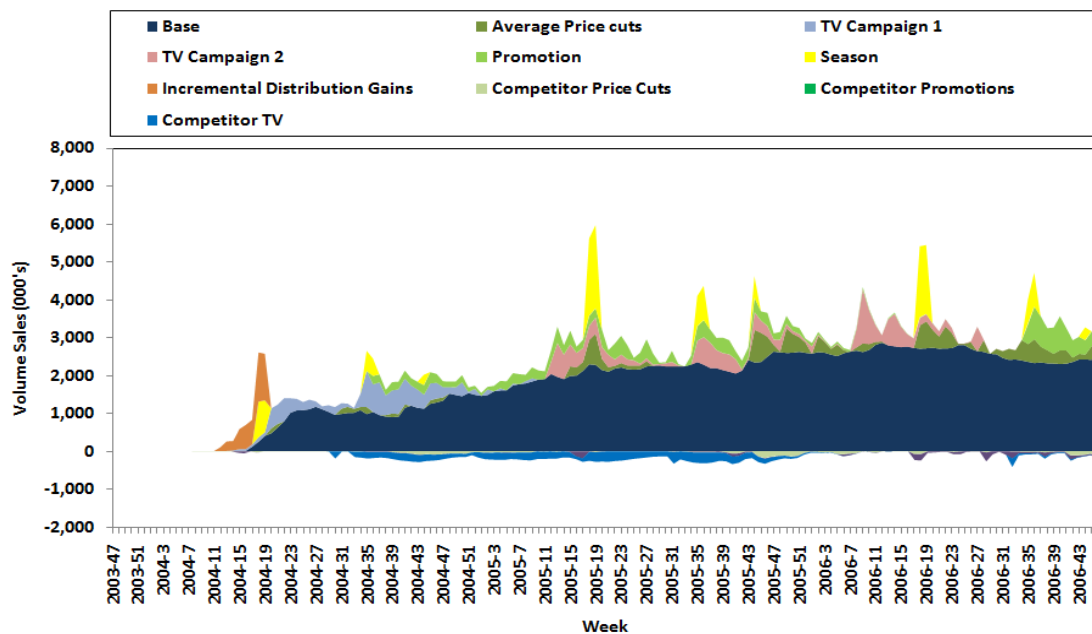
*Source: Cain (2008)

A large proportion of incremental sales volume is driven by TV advertising over the period – giving an average uplift of 6% over base sales. Whereas this is reasonable for

established brands in the industry - and consistent with decent cut through as measured by advertising awareness scores – the early heavyweight launch campaign delivers below average return for a new product in the market, bringing down the overall short-term TV ROI. Seasonal demand, the initial distribution drive and increases in in-store presence over the sample all contribute a significant percentage of incremental volume. Promotional activity, in the form of multi-buys, also play a significant role – yet pure price cuts drive little volume. Competitor losses - representing the potential sales volume the brand could have achieved had the competitors not engaged in those activities - amounts to an average of approximately 5% of total sales volume over the sample - with the bulk of lost volume due to competitor TV media.

4.2) Category approach and the relative view

Decompositions of absolute sales volume are an important part of the process, but only tell part of the story. To provide additional direction on marketing strategies, manufacturers often wish to know whether reported incremental marketing effectiveness is good or bad in the context of the overall category. To answer this question, a set of benchmarks is needed. The usual approach is to appeal to historical studies of similar brands in the category. However, these are not like-for-like comparisons since we are comparing studies at different points in time, at different stages of the product lifecycle(s) and with each study potentially subject to a whole host of different influencing factors. The category modelling approach outlined in Section 1, on the other hand, provides a set of directly comparable benchmarks over the same time period. These come in two stages.

The first stage provides an estimate of the overall net performance of the brand's marketing strategy relative to all other brands in the category. This is illustrated in Figure 5, where the model presented in Figure 4 has been re-estimated as part of a category system of equations using the dynamic category model analogue of equations 2(a)-2(e).

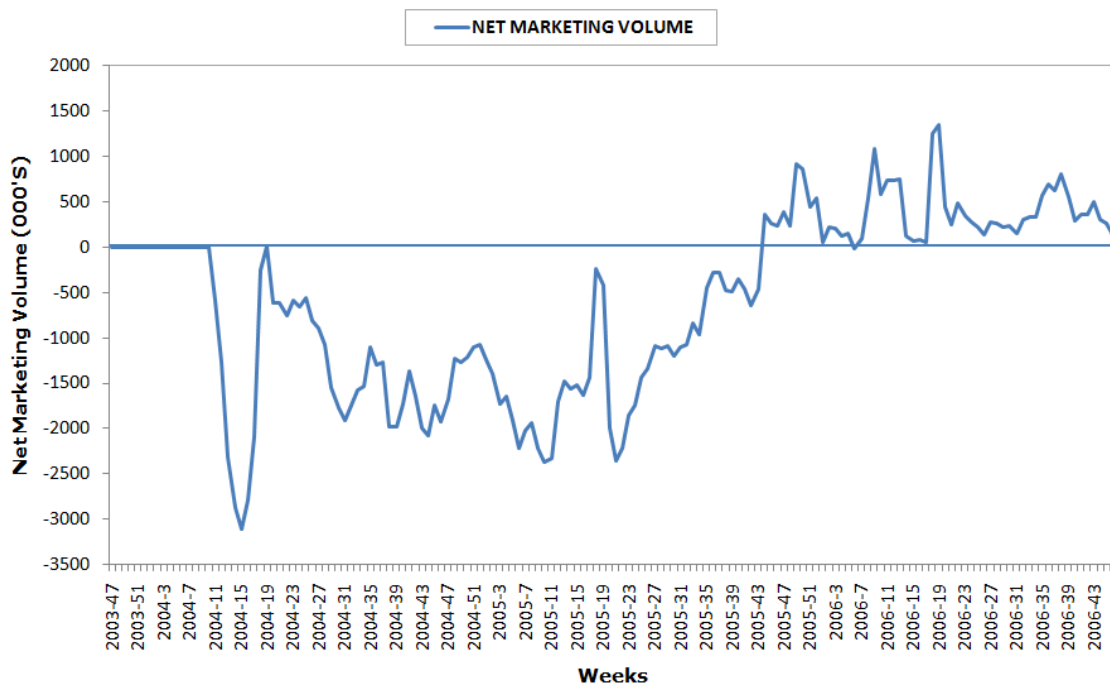**Figure 5: Relative brand marketing volume**

Figure 5 gives the brand manager a bird's eye view of its net marketing performance in the context of the whole category: values above and below the horizontal axis are indicative of a performance above and below the category average respectively. Here we can see that the client's brand marketing is underperforming for much of the sample. It is natural to want to know which element(s) of the marketing mix are driving this inefficiency.

The second stage answers this question by identifying the contribution of each individual marketing mechanic to the over or under-performance of the overall marketing strategy. This is illustrated in Figure 6, which simply presents an alternative view of the model output to that in Figure 4. Each element of marketing volume for the client brand is now quantified relative to all other brands in the category.

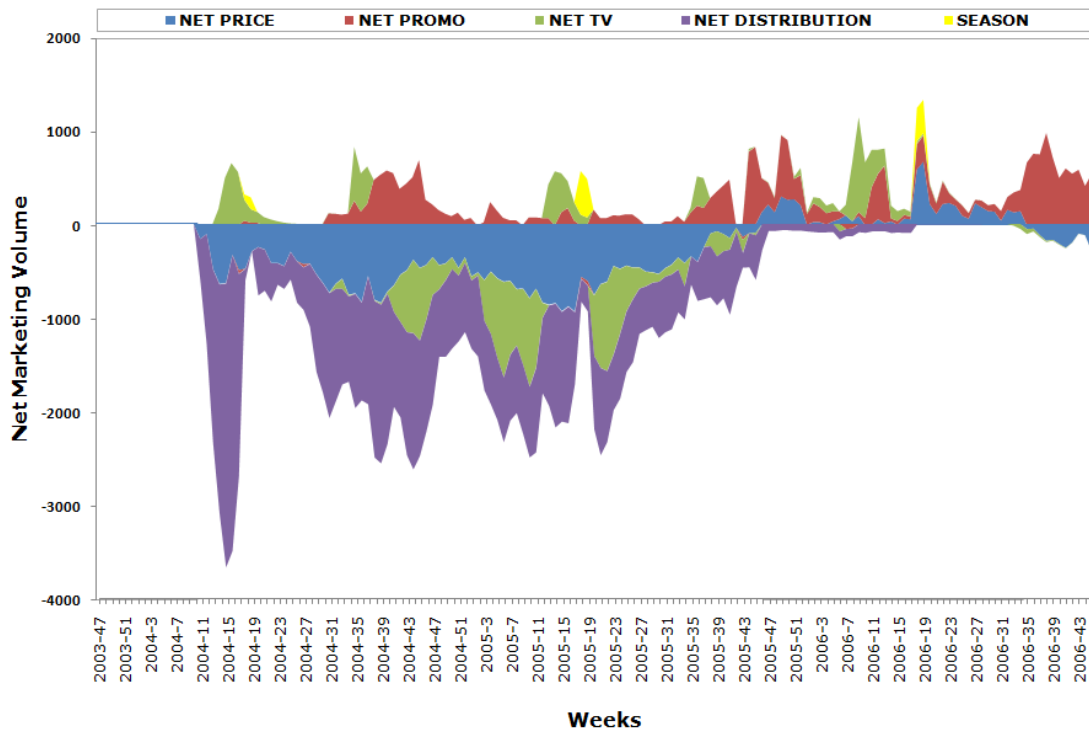**Figure 6: Drivers of relative brand marketing volume**

Figure 6 provides valuable additional insights that are lost to standard single-equation studies of brand sales volume. Whereas absolute short-term TV media return is a reasonable 6% uplift over base, in relative terms the client brand is losing volume due to its advertising strategy over a significant part of the sample (2004 week 40 through to 2005 week 33). This highlights a relatively weak media strategy – otherwise hidden if we concentrate on absolute volumes alone. A similar story emerges for selling distribution, where respectable absolute incremental volume gains hide a weak relative position – particularly during the launch drive. This helps to explain the poor absolute volume returns for the TV launch campaign. The poor average price position echoes the poor absolute price cut return evident in Figure 4: the brand's average price position in the category needs to be reviewed. The favourable promotional picture of Figure 4 is preserved in the relative view: the brand's promotional positioning is pitched correctly.

5) Indirect marketing effects and total ROI

Short-term ROI, whether viewed in absolute or relative form, is only part of the story. Marketing investments do more than simply drive incremental sales volumes. In the first place, successful TV campaigns serve to build trial, stimulate repeat purchase and maintain healthy consumer brand perceptions. In this way, advertising can drive and sustain the level of brand base sales.[19] Secondly, advertising can affect the degree of product price sensitivity – thereby enabling the brand to command higher price premia. Only by quantifying such indirect effects can we evaluate the true ROI to marketing investments and arrive at an optimal strategic balance between them.

Estimation of indirect effects requires four key data inputs: marketing investments, brand perceptions, base sales and price elasticity evolution. Base sales evolution indicates the extent to which new purchasers are converted into loyal consumers – through persistent repeat purchase behaviour and lasting shifts in consumer product tastes. This, in turn, can lead to shifts in price elasticity as stronger equity reduces demand sensitivity to price change. Brand perceptions are forged by product experience, driving product tastes and repeat purchase behaviour. Marketing investments, in turn, work directly on product perceptions. This reasoning creates the flow illustrated in Figure 7, where marketing investments are linked to variation in base sales and price elasticity via brand perception data. Given the evolutionary nature of the base sales (and other) data involved, the appropriate estimation process follows the five key stages outlined in sections 5.1 – 5.5 below.[20]

---

[19]Conversely, excessive price promotional activity can negatively influence base sales evolution – via denigrating brand perceptions and stemming repeat purchase.

[20] Note that for any long-term or permanent indirect brand-building effects to exist, brand sales must be evolving. This is true if Model 2(a) encompasses the conventional mix model – with base sales representing the evolutionary component. The flow illustrated in Figure 7 is often referred to as a Path Model and estimated using Structural Equation Modelling (SEM) techniques. However, conventional SEM analysis is not suitable for evolving or non-stationary data in levels.

Figure 7: The indirect effects of marketing
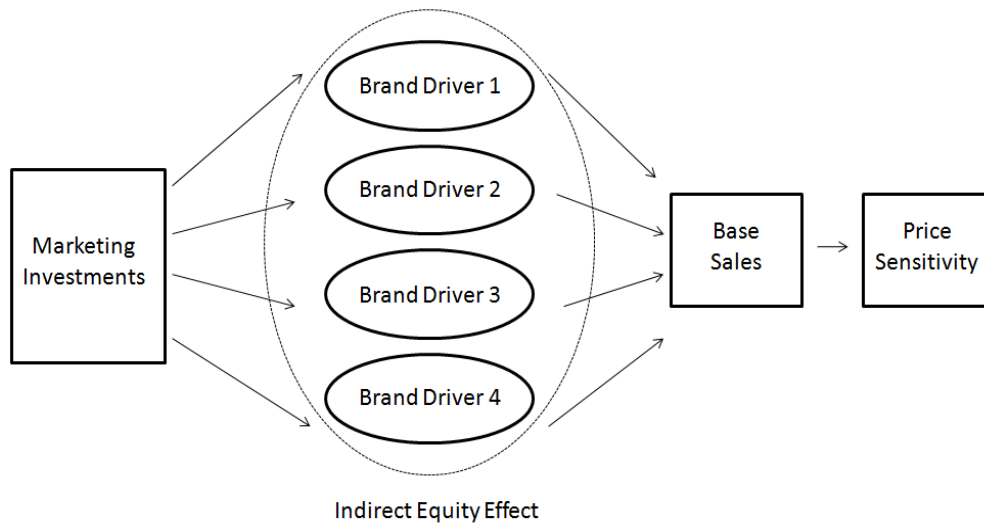
5.1) Estimating evolution in base sales and price sensitivity

Evolution in base sales and price sensitivity can be derived directly from the time series approach to the marketing mix model outlined in Section 3.2. For example, base sales from the face cleansing product model presented in Section 4, together with estimated variation in average price elasticity, are illustrated in Figure 8. Price sensitivity falls from -1.80 at the beginning of the sample to -1.30 at the beginning of 2006 - in line with a rising loyal consumer base after product launch. Price sensitivity rises thereafter to approximately -1.40 by the end of the sample.

Figure 8: Time series evolution in bases sales and price elasticity
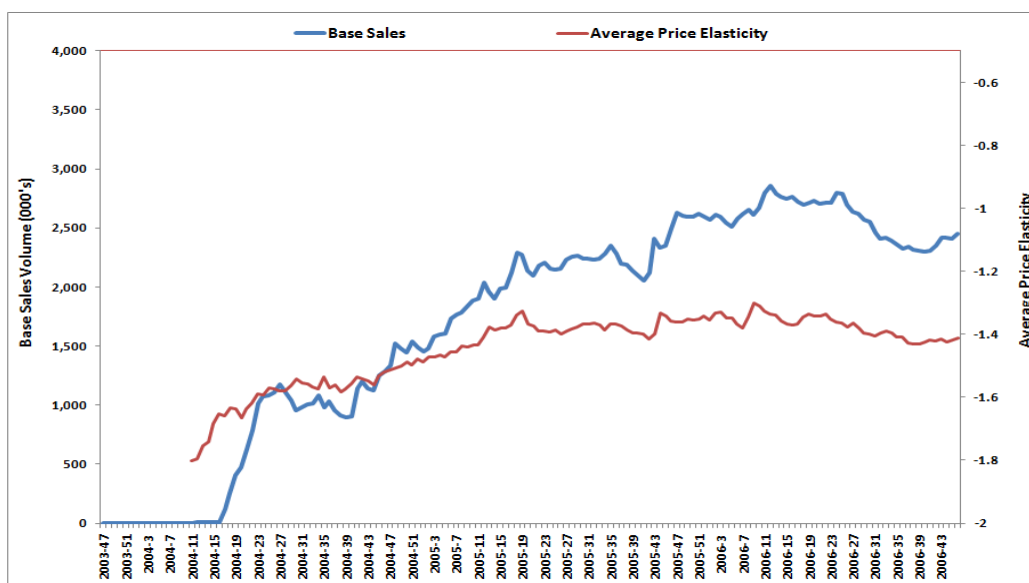
5.2) Identifying relevant consumer brand perceptions

Secondly, important consumer beliefs or attitudes towards the brand are identified. These will encompass statements about the product, perception of its value, quality and image. Such data are routinely supplied by primary consumer research tracking companies. Data are usually recorded weekly over time - often rolled up into four weekly moving average time series to minimise the influence of sampling error. An example is illustrated in Figure 9 below, which plots the evolving baseline of Figure 4 alongside advertising TVR investments and brand perception data relating to fragrance and perceived product value.[21]

Figure 9: Evolution of base and consumer tracking statements*
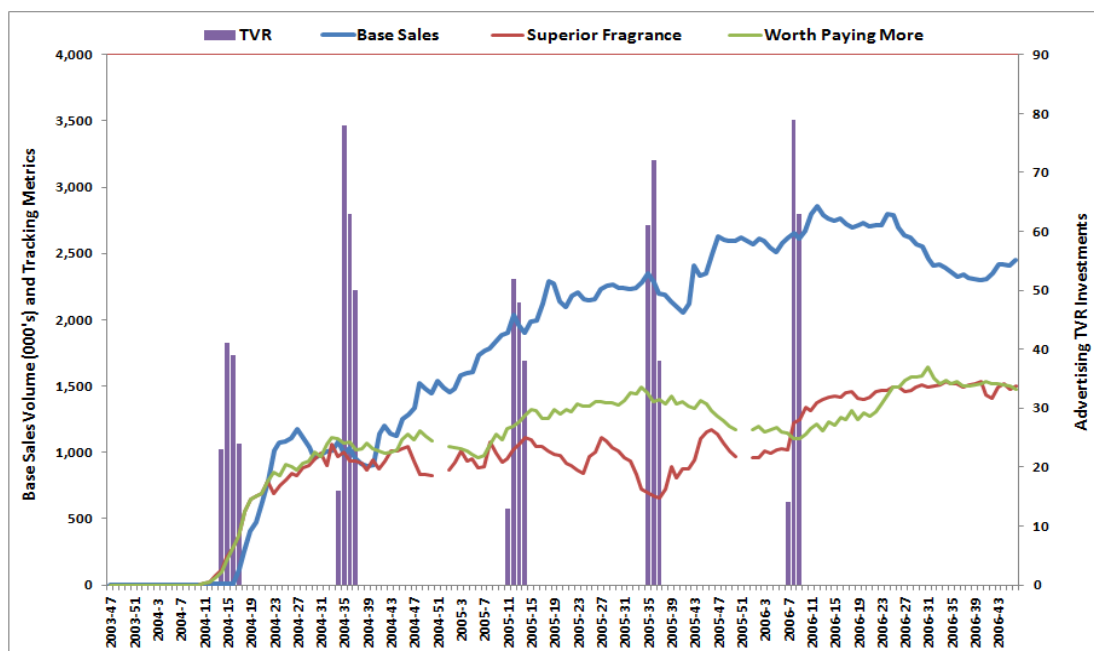
*Source: Cain (2008)

---

[21] Selected brand image statements are often highly collinear. Consequently, preliminary *factoring* analysis is usually undertaken to separate out the data into mutually exclusive themes or groups prior to modelling.

5.3) Contribution of brand perceptions to brand demand and price sensitivity

Thirdly, we establish the impact of relevant tracking measures on brand demand and price sensitivity. Brand image tracking data represent the variation in consumer brand perceptions over time. Extracted base sales represent evolution of observed brand purchases or long-run brand demand - driven by trends in shelf price, selling distribution and, crucially, brand perceptions.[22] Regression analysis is used to identify relationships between these variables. When evolving variables are involved, we must be careful to avoid spurious correlations where the analysis is simply picking up unrelated trending activity. Only then can we interpret the regression coefficients as valid estimates of the importance of each of the base demand drivers. The cointegrated Vector Auto Regression (VAR) model (Johansen, 1996, Juselius, 2006) is used for this purpose and demonstrated with the following model structure.

$$
\begin{bmatrix} \Delta X_{1t} \\ \Delta X_{2t} \\ \Delta X_{3t} \\ \Delta X_{4t} \\ \Delta X_{5t} \\ \Delta X_{6t} \\ \Delta X_{7t} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & \pi_{15} & \pi_{16} & \pi_{17} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} & \pi_{25} & \pi_{26} & \pi_{27} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} & \pi_{35} & \pi_{36} & \pi_{37} \\ \pi_{41} & \pi_{42} & \pi_{43} & \pi_{44} & \pi_{45} & \pi_{46} & \pi_{47} \\ \pi_{51} & \pi_{52} & \pi_{53} & \pi_{54} & \pi_{55} & \pi_{56} & \pi_{57} \\ \pi_{61} & \pi_{62} & \pi_{63} & \pi_{64} & \pi_{65} & \pi_{66} & \pi_{67} \\ \pi_{71} & \pi_{72} & \pi_{73} & \pi_{74} & \pi_{75} & \pi_{76} & \pi_{77} \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \\ X_{5t-1} \\ X_{6t-1} \\ X_{7t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \\ \varepsilon_{6t} \\ \varepsilon_{7t} \end{bmatrix} \quad (15)
$$

Equation (15) represents an unrestricted VAR model, the multivariate analogue of equation (13), re-parameterised as a Vector Error Correction Model (VECM), the multivariate analogue of equation (14).[23] Variables X_{1t} - X_{7t} represent base sales, average price elasticity evolution, regular shelf price, selling distribution, two image statements and advertising data.[24] Model (15) is first used to test for *equilibrium*

---

[22] Average price and distribution contributions illustrated in Figure 4 are incremental – expressed as short-term deviations from long-term trend. Long-term price and distribution trends are absorbed into the baseline during the decomposition process. It is these trends that are used in model (15).

[23] Whereas the VAR technique is impractical in the context of the fully specified mix model due to the large number of variables generally involved, the focus on base sales evolution allows us to concentrate on a small group of variables, greatly simplifying the approach. Model (15) is derived from a VAR(1) specification – where all the variables appear with a one-period lag. The appropriate number of lags is generally tested such that each equation depicts a statistically congruent representation of the data.

[24] Advertising data often comes in the form of TVR 'bursts' as illustrated in Figure 9. Under these circumstances, given the discrete nature of such data, it cannot be modelled as an endogenous variable in the system. Under these circumstances we would use (continuous) adstocked TVR data in (15) and condition on this (weakly exogenous) variable in estimation. Alternatively, we would transform the TVR data into a continuous Total Brand Communication Awareness variable.

relationships between the variables: that is, relationships which tend to be restored when disturbed such that the series follow long-run paths together over time. Conceptually, this occurs if linear combinations of the variables provide trendless (stationary) relationships, implying that the π matrix of equation (15) is of reduced rank and the variables cointegrate. With n trending I(1) variables, the π matrix may be up to rank $n\text{-}1$, with $n\text{-}1$ corresponding equilibrium relationships to be tested as part of the model process.[25] For ease of exposition - and since we are focusing primarily on the drivers of base sales and price sensitivity - we assume a rank of 2 and thus just two linearly independent cointegrating relationships. This allows us to factorise (15) as:

$$
\begin{bmatrix} \Delta X_{1t} \\ \Delta X_{2t} \\ \Delta X_{3t} \\ \Delta X_{4t} \\ \Delta X_{5t} \\ \Delta X_{6t} \\ \Delta X_{7t} \end{bmatrix} = \begin{bmatrix} \alpha_{11}\alpha_{12}\,\alpha_{13} \\ \alpha_{21}\alpha_{22}\,\alpha_{23} \\ \alpha_{31}\alpha_{32}\,\alpha_{33} \\ \alpha_{41}\alpha_{42}\,\alpha_{43} \\ \alpha_{51}\alpha_{52}\,\alpha_{53} \\ \alpha_{61}\alpha_{62}\,\alpha_{63} \\ \alpha_{71}\alpha_{72}\,\alpha_{73} \end{bmatrix} \begin{bmatrix} \beta_{11} & 0 & \beta_{31} & \beta_{41} & \beta_{51} & \beta_{61} & 0 \\ \beta_{12} & \beta_{22} & 0 & 0 & \beta_{52} & \beta_{62} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_{73} \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \\ X_{5t-1} \\ X_{6t-1} \\ X_{7t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \\ \varepsilon_{6t} \\ \varepsilon_{7t} \end{bmatrix} \quad (15a)
$$

Equation 15(a) represents a cointegrated VAR representation of the system − with each first differenced equation driven by (stationary) advertising investments and two cointegrating or equilibrium relationships between base sales, average price elasticity, regular price evolution, selling distribution and the two image statements. The parameters $\beta_{11} - \beta_{61}$ and $\beta_{12} - \beta_{62}$ represent the cointegrating parameters. If we take the first cointegrating vector, and normalise on base sales (X_1) by setting β_{11} to unity, then β_{31}, β_{41} β_{51} and β_{61} represent the impact of the regular price level, selling distribution and the two image statements on base sales.[26] If we then take the second cointegrating vector and normalise on price elasticity (X_2) by setting β_{22} to unity, then β_{12}, β_{52} and β_{62} represent the impact of base sales and the two image statements on price

---

[25]To provide valid cointegrating relationships with base sales, other variables such as regular price, distribution and image statements must also be evolving. Advertising is generally stationary and would not enter the cointegrating relationship, reflected by the zero entries in the last column of the beta matrix above. The variable itself thus represents a stationary 'combination' and is represented by the third row in the beta matrix with $n\text{-}1$ restrictions, normalised on β_{73}.

[26]Note that the regular price parameter estimate is distinct from the average price elasticity derived from the short-term mix model.

sensitivity. Additional identifying constraints can be placed on the vectors. For example, we would expect base sales evolution to drive average price sensitivity – as per the flow illustrated in Figure 7 - but not vice versa. Thus we would set β_{21} to zero in the first cointegrating relationship. Furthermore, unless we have reason to believe that the level of regular price and selling distribution influences average price sensitivity, we would set β_{32} and β_{42} to zero in the second cointegrating vector.

Normalisation restrictions are quite arbitrary – and reflect assumptions on which variables are adjusting in the system: that is, the endogenous variables and direction of causality. For example, by normalising on X_1 and X_2 in each of the cointegrating vectors, we pre-suppose that image statements drive base sales and price sensitivity. However, it may be that causality runs in the other, or both, directions. The significance of the parameters α_{11}, α_{51} and α_{61} in the equations for ΔX_1, ΔX_5 and ΔX_6 provide the relevant information for base sales. Suppose α_{11} is negative and significant in the equation for ΔX_1, yet α_{51} and α_{61} are zero in equations ΔX_5 and ΔX_6. This tells us that base sales adjust (error correct) to shifts in image statements X_5 and X_6, at a rate α_{11} weighted by β_{51} and β_{61} respectively. However, image statements do not adjust to movements in base sales. Brand perceptions are (weakly) exogenous and *Granger* cause base sales (Granger, 1987). However, if α_{51} and α_{61} are positive and significant in equations for ΔX_5 and ΔX_6 then image statements do adjust to movements in base sales. Causality is bi-directional: from image to base and vice versa. Similar reasoning applies to the equation for ΔX_2, where, for a causal relationship from image statements to price sensitivity, we would expect α_{22} to be negative and significant with α_{52} and α_{62} equal to zero in the equations for ΔX_5 and ΔX_6. A negative and significant estimate of α_{12} would also tell us that base sales Granger cause price sensitivity.

5.4) Linking marketing investments to base sales

Finally, model 15(a) is used to estimate the full (long-term) impact of advertising on brand perceptions and the impact of the latter on base sales and price sensitivity. To do this, we make use of the Moving Average representation of the cointegrated VAR model 16(a) – written in matrix form as follows:

$$X_t = A + C\sum_{i=0}^{t} \varepsilon_i + \sum_{i=0}^{\infty} C_i^* \varepsilon_{t-i} \tag{16}$$

Equation (16) shows that the model can be broken down into three components: initial starting values (*A*) for the variables, a non-stationary permanent component and a stationary component – represented by the cointegrating vectors themselves. The non-stationary C matrix – known as the Moving Average impact matrix – is illustrated in Figure 10 and provides the long-term impact of base sales on price sensitivity, image statements on base sales and advertising on image statements: each may then be combined to predict the net indirect impact of advertising on base sales and price sensitivity.

Figure 10: Moving Average Impact Matrix

	ε_{X1}	ε_{X2}	ε_{X3}	ε_{X4}	ε_{X5}	ε_{X6}	ε_{X7}
X_1	C_{11}	C_{12}	C_{13}	C_{16}	$\mathbf{C_{15}}$	$\mathbf{C_{16}}$	0
X_2	$\mathbf{C_{21}}$	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}
X_3	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}	C_{37}
X_4	C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}	C_{47}
X_5	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	C_{56}	$\mathbf{C_{57}}$
X_6	C_{61}	C_{62}	C_{63}	C_{64}	C_{65}	C_{66}	$\mathbf{C_{67}}$
X_7	0	0	0	0	0	0	0

Each column of Figure (10) represents the cumulated empirical shocks to each equation of the VECM system 15(a).[27] Reading across in rows, the parameters indicate the long-term (permanent) impact of such cumulated shocks on the levels of the variables in the system. For example, the first row indicates that the long-term

---

[27] Note that shocks have to be identified as *structural* to ensure that they derive from the variable of interest and are not contaminated by effects from other variables in the system (see *inter alia* Juselius, 2006).

behaviour of X_1 is determined by shocks in $X_2 - X_6$ with weights C_{12} -C_{16}. Shocks in X_7 have no direct impact since base sales do not contain any of the direct impact of TV advertising by construction.[28] The final row is populated with zeros, indicating that shocks of all variables in the system have no long-term impact on advertising. This follows by construction since X_7 is a stationary variable.

For the permanent indirect impacts of advertising on base sales, the parameters of interest are C_{15}, C_{16}, C_{57} and C_{67}. The latter two parameters measure the impact of advertising shocks on image statements X_5 and X_6 respectively. Parameters C_{15} and C_{16} on the other hand measure the impact of shocks in image statements X_5 and X_6 on base sales. The net indirect impact of 1% changes in advertising and both image statements on base sales is, therefore, %$\{(C_{15}* C_{57}) + (C_{16}* C_{67})\}$. The impact of base sales on price sensitivity is given by parameter C_{21}. The impact of advertising on base sales is thus augmented with %$C_{21}*[\%\{(C_{15}* C_{57}) + (C_{16}* C_{67})\}]$ to incorporate the impact of advertising on price sensitivity.[29]

5.5) Calculating the full long-run impact

Estimated baseline impacts of marketing investments are part of the long-run sales trend and as such generate a stream of effects extending into the foreseeable future: positive for TV advertising and (potentially) negative for heavy promotional weight. These must be quantified if we wish to measure the full extent of such effects. To do so, we first note that in practice we would not expect future benefit streams to persist indefinitely into the future. Various factors dictate that such benefits will decay over time. Firstly, the value of each subsequent period's impact will diminish as loyal consumers eventually leave the category and/or switch to competing brands. Secondly, future benefits will be worth less as uncertainty increases. To capture these effects, we exploit a standard discounting method used in financial accounting which

---

[28] The underlying sales trend in equation 2(b) implies that marketing effects are orthogonal to the baseline. Thus, direct long-term marketing effects are zero by construction and any long-term effects are indirect – working through brand perceptions. An alternative approach, as discussed in Cain (2005) and exemplified in Osinga *et al* (2009) is to specify the trend transition equation directly as a function of marketing effects thus allowing endogenous trend evolution.

[29] Note that significant MA impact coefficients imply permanent or *hysteretic* indirect effects. Non-significant or zero MA coefficients do not, however, imply zero indirect effects. Even though the impulse response functions may decay to zero in the limit, any short to medium-term impulse effects are still evidence of indirect marketing effects – in addition to those measured in the short-term model.

quantifies the current value of future revenue streams. The calculation used for each marketing investment is written as:

$$PV_i = \sum_{t=1}^{N} \frac{C_{i0} * d_i^{\,t}}{(1+r)^t} \tag{18}$$

Where PV_i denotes the Present Value of future indirect revenues accruing to marketing investment i, C_{i0} represents the indirect benefit calculated over the model sample, d represents the per period decay rate of subsequent indirect revenues over N periods and r represents a discount rate reflecting increasing uncertainty. The final PV of indirect marketing revenue streams will depend critically on the chosen values of d and r. The benefit decay rate can be chosen on the basis of established norms or estimated from historical data. The discount rate is chosen to reflect the product manufacturer's internal rate of return on capital: a higher discount rate reflects greater uncertainty around future revenue streams.

The indirect 'base-shifting' impact over the model sample, together with the decayed *PV* of future revenue streams quantifies the long-run base impact of advertising and promotional investments. The value created by the impact of advertising on price elasticity, on the other hand, derives from the fact that the brand can now charge a higher price for the same quantity with less impact on marginal revenue. The reduced impact of price increases on revenues, weighted by the advertising contribution to price elasticity evolution, provides the additional value impact of advertising. Both the base and price elasticity revenue effects may then be combined with the weekly revenues calculated from the short-run modelling process. Benchmarking final net revenues against initial outlays then allows calculation of a more holistic ROI to marketing investments.[30]

---

[30] Note that TV investments may serve to simply maintain base sales – with no observable impact picked up using time series econometric modelling. This can be dealt with by incorporating estimates of base decay in the absence of advertising investments – based on prior 'norms' or 'meta' analyses across similar brands in similar categories. Note also, that excessive price promotion may serve to increase price sensitivity by changing the consumer's price reference point. This constitutes an additional negative impact of price promotions on net revenues.

C. Managerial benefits

The benefits and uses of the conventional marketing mix model are extensive. As a minimum, model results are used to provide ROI measurement and a purely retrospective view of marketing performance: that is, what worked and what didn't. This step may then be augmented in two key ways. In the first place, elasticity estimates can be used to advise on optimal allocation of the marketing budget. Secondly, marketing elasticities allow simulation of the likely consequences of alternative mix scenarios – both retrospectively and from a forward looking perspective if future marketing plans are available. This leads naturally onto the use of the model for short to medium-term forecasting.

Re-formulating the conventional mix model to explicitly capture both short and long-term sales variation takes us a step further. Not only does it provide a more accurate set of deliverables but lays the foundation of the long-run modelling processes outlined in Section B. This, in turn, delivers two key commercial benefits.

i) Enhanced strategic budget allocation

Budget allocation decisions based on short-term mix model results are short-run focused by construction, often favouring heavy promotional activity over media investments. This often leads to a denigration of brand equity in favour of short-run revenue gain. Incorporating long-run returns, however, provides a more holistic balance reflecting strategic brand-building media activity.

ii) Enhanced media creative building

A key part of the flow illustrated in Figure 7 is the equilibrium relationship between consumer brand perceptions, base sales and price elasticity. This relationship can be used to test for causal links between the data. Understanding which key brand characteristics drive brand demand and price elasticity in this way can help to clarify the media creative process for more effective long-term brand strategy.

Neither strategic benefit is possible if we rely on conventional market mix modelling alone, illustrating the additional insights that can be obtained from a combination of primary consumer research and modern time series analysis of secondary source sales and business data.

D. Conclusions

This chapter has sought to illustrate the complete structure of the marketing mix modelling process, ranging from the microeconomic consumer theory underlying alternative functional forms, dynamic estimating specifications through to models of long-term consumer behaviour. Throughout, three key issues have been demonstrated.

Firstly, all marketing mix models are rooted in economic theories of the consumer – and as such appropriate theoretical models need to be chosen for each market, category and business issue to hand. Practical applications of the mix model tend to routinely apply the standard single equation double logarithmic functional form regardless of circumstance. This is not necessarily the best route and superior models can be developed depending on the data available.

Secondly, dynamic form and estimation technique are crucial. Conventional dynamic marketing mix models only go so far and the estimation structures used are inadequate for modelling time series data, where it is imperative to correctly deal with the season and evolving trend or baseline inherent in most economic time series. A preferable and more flexible approach is to re-structure the mix model to explicitly model both the short and long-run features of the data.

Finally, not only does the time series structure provide more accurate short-run marketing results but, when combined with evolution in intermediate brand perception measures, allows for an evaluation of the long-run impact of marketing activities. This illustrates how intermediate brand perception data can be shown to (Granger) cause brand sales and, contrary to the concerns raised by Binet *et al* (2007), used to improve long-term business performance.

References

Almon, S, "The Distributed Lag Between Capital Appropriations and Expenditures" *Econometrica* 33: pp. 178-196, January 1965.

Baghestani, H. (1991), "Cointegration analysis of the advertising-sales relationship" *Journal of Industrial Economics*, Vol. 39 No.6, pp.671-81.

Barten, A.P. (1966), *Theorie en empirie van een volledig stelsel van vraagvergelijkingen,* Doctoral dissertation, Rotterdam: University of Rotterdam.

Barten, A.P. (1969), "Maximum Likelihood estimation of a complete system of demand equations," *European Economic Review*, Vol. 1, pp 7-73.

Binet, L. and Field, P. (2007) 'Marketing in the Era of Accountability', *IPA Datamine, World Advertising Research Centre*, London.

Broadbent, S. (1979) "One Way TV Advertisements Work", Journal of the Market Research Society Vol. 23 no.3.

Cain, P.M. (2005), 'Modelling and forecasting brand share: a dynamic demand system approach', *International Journal of Research in Marketing*, 22, 203-20.

Cain, P.M. (2008), 'Limitations of conventional market mix modelling', *Admap, April* pp 48-51

Clarke, Darral G. (1976), "Econometric Measurement of the duration of Advertising Effect on Sales", *Journal of Marketing Research*, 13, 345-57.

Deaton, A. and Muellbauer, J. (1980), "Economics and consumer behavior", *Cambridge University Press, Cambridge.*

Dekimpe, M. and Hanssens, D. (1995), "Empirical generalisations about market evolution and stationarity", *Marketing Science,* 14, (3): G109-G121.

Dekimpe, M. and Hanssens, D. (1999), "Sustained spending and persistent response: a new look at long-term marketing profitability", *Journal of Marketing Research*, 36, pp 397-412

Dickey, D.A. and Fuller, W.A. (1981), "Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica*, 49, 1057-72.

Givon, M. and Horsky, D. (1990), "Untangling the effects of Purchase Reinforcement and Advertising Carryover", *Marketing Science*, 9:2 (Spring), 171-87.

Granger, C.W.J,(1988), "Some recent development in a concept of causality", *Journal of Econometrics*, (39), 199-211

Harvey, A.C. (1989), "Forecasting, structural time series models and the Kalman Filter", *Cambridge University Press, Cambridge.*

Houston, F.S., Kanetkar, V. Weiss, D.L (1992), "Estimation procedures for MCI and MNL models: a comparison of reduced forms" Unpublished working paper, University of Toronto, Ontario.

Jedidi, K., Mela, C. and Gupta, S. (1999), "Managing advertising and promotion for long-run profitability", *Marketing Science* 181-22.

Johansen, S. (1996). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, 2.edn.* Advanced Texts in Econometrics, Oxford University Press: Oxford.

Juselius, K. (2006), The *Cointegrated VAR model: Methodology and Applications.* Advanced Texts in Econometrics. Oxford University Press: Oxford.

Koyck, L.M. (1954), *Distributed Lags and Investment Analysis.* Amsterdam: North Holland.

Mela, C., Gupta, S. and Lehmann, D. (1997), "The long-term impact of promotion and advertising on consumer brand choice", *Journal of Marketing* Research 34 (2), pp 248-261.

Nakanishi, Masao and Lee G. Cooper (1974), "Parameter Estimation for a Multiplicative Competitive Interaction Least Squares Approach", *Journal of Marketing Research*, 11, 303-11.

Osinga, Ernst,C, Leeflang, Peter, S.H, and Wieringa, Jaap.E (2009), "Early Marketing Matters: a time varying parameter approach to persistence modelling", Journal of Marketing Research, Vol. XLV1.

Schwert, G. William. (1987), "Effects of model specification on tests for unit roots in Macroeconomic data", *Journal of Monetary Economics*, Elsevier, vol. 20(1), pp 73-103.

Stone, J.R.N (1954), "Linear Expenditure Systems and demand analysis: an application to the pattern of British demand", *Economic Journal*, Vol. 64, pp. 511-27.

Theil, H. (1965), "The Information Approach to demand analysis", *Econometrica*, Vol. 33, pp. 67-87.